

$f_x(2, 1)$ is the rate of change of f at $(2, 1)$ in the x -direction. If we start at $(2, 1)$, where $f(2, 1) = 10$, and move in the positive x -direction, we reach the next contour line (where $f(x, y) = 12$) after approximately 0.6 units. This represents an average rate of change of about $\frac{2}{0.6}$. If we approach the point $(2, 1)$ from the left (moving in the positive x -direction) the output values increase from 8 to 10 with an increase in x of approximately 0.9 units, corresponding to an average rate of change of $\frac{2}{0.9}$. A good estimate for $f_x(2, 1)$ would be the average of these two, so $f_x(2, 1) \approx 2.8$. Similarly, $f_y(2, 1)$ is the rate of change of f at $(2, 1)$ in the y -direction. If we approach $(2, 1)$ from below, the output values decrease from 12 to 10 with a change in y of approximately 1 unit, corresponding to an average rate of change of -2 . If we start at $(2, 1)$ and move in the positive y -direction, the output values decrease from 10 to 8 after approximately 0.9 units, a rate of change of $\frac{-2}{0.9}$. Averaging these two results, we estimate $f_y(2, 1) \approx -2.1$.