



$$a = 2$$

$$b = 6$$

- (a) $C_1: x = t \Rightarrow dx = dt, y = 0 \Rightarrow dy = 0 dt, 0 \leq t \leq 2.$
 $C_2: x = 2 \Rightarrow dx = 0 dt, y = t \Rightarrow dy = dt, 0 \leq t \leq 6.$
 $C_3: x = 2 - t \Rightarrow dx = -dt, y = 6 - 3t \Rightarrow dy = -3 dt, 0 \leq t \leq 2.$

Thus

$$\begin{aligned} \oint_C xy \, dx + x^2 y^3 \, dy &= \oint_{C_1+C_2+C_3} xy \, dx + x^2 y^3 \, dy \\ &= \int_0^2 0 \, dt + \int_0^6 4t^3 \, dt \\ &\quad + \int_0^2 [-(2-t)(6-3t) - 3(2-t)^2(6-3t)^3] \, dt \\ &= 0 + [t^4]_0^6 + [(2-t)^3 + \frac{27}{2}(2-t)^6]_0^2 \\ &= 1296 - 872 = 424 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad \oint_C xy \, dx + x^2 y^3 \, dy &= \iint_D \left[\frac{\partial}{\partial x} (x^2 y^3) - \frac{\partial}{\partial y} (xy) \right] dA \\ &= \int_0^2 \int_0^{3x} (2xy^3 - x) \, dy \, dx = \int_0^2 \left[\frac{1}{2}xy^4 - xy \right]_{y=0}^{y=3x} dx \\ &= \int_0^2 \left(\frac{81}{2}x^5 - 3x^2 \right) dx = 432 - 8 = 424 \end{aligned}$$