

The boundary curve C is the circle in the yz -plane. By Equation 3, $\iint_{S_1} \text{curl } \mathbf{F} \cdot d\mathbf{S} = \oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_{S_2} \text{curl } \mathbf{F} \cdot d\mathbf{S}$ where S_1 is the original hemisphere and S_2 is the disk $y^2 + z^2 \leq 36$, $x = 0$.

$\text{curl } \mathbf{F} = (x - x^2)\mathbf{i} - (y + e^{xy} \sin z)\mathbf{j} + (2xz - xe^{xy} \cos z)\mathbf{k}$, and for S_2 we choose $\mathbf{n} = \mathbf{i}$ so that C has the same orientation for both surfaces. Then $\text{curl } \mathbf{F} \cdot \mathbf{n} = x - x^2$ on S_2 , where $x = 0$.

$$\text{Thus } \iint_{S_2} \text{curl } \mathbf{F} \cdot d\mathbf{S} = \iint_{y^2+z^2 \leq 36} (x - x^2) dA = \iint_{y^2+z^2 \leq 36} 0 dA = 0.$$

Alternatively, we can evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$: C with positive orientation is given by $\mathbf{r}(t) = \langle 0, 6 \cos t, 6 \sin t \rangle$, $0 \leq t \leq 2\pi$, and

$$\begin{aligned} \iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S} &= \oint_C \mathbf{F} \cdot d\mathbf{r} \\ &= \int_0^{2\pi} \langle e^{0(6 \cos t)} \cos(6 \sin t), (0)^2 (6 \sin t), (0)(6 \cos t) \rangle \\ &\quad \cdot \langle 0, -6 \sin t, 6 \cos t \rangle dt \\ &= \int_0^{2\pi} 0 dt = 0. \end{aligned}$$