

$\operatorname{div} \mathbf{F} = 4 + x + 3x = 4 + 4x$, so

$\iiint_E \operatorname{div} \mathbf{F} \, dV = \int_0^2 \int_0^2 \int_0^2 (4x + 4) \, dx \, dy \, dz = 64$ (notice the triple integral is **four** times the volume of the cube plus **four** times \bar{x}).

To compute $\iint_S \mathbf{F} \cdot d\mathbf{S}$, on

$$S_1: \mathbf{n} = \mathbf{i}, \mathbf{F} = 8\mathbf{i} + 2y\mathbf{j} + 6z\mathbf{k}, \text{ and } \iint_{S_1} \mathbf{F} \cdot d\mathbf{S} = \iint_{S_1} 8 \, dS = 32;$$

$$S_2: \mathbf{F} = 4x\mathbf{i} + 2x\mathbf{j} + 3xz\mathbf{k}, \mathbf{n} = \mathbf{j} \text{ and } \iint_{S_2} \mathbf{F} \cdot d\mathbf{S} = \iint_{S_2} 2x \, dS = 8;$$

$$S_3: \mathbf{F} = 4x\mathbf{i} + xy\mathbf{j} + 6x\mathbf{k}, \mathbf{n} = \mathbf{k} \text{ and } \iint_{S_3} \mathbf{F} \cdot d\mathbf{S} = \iint_{S_3} 6x \, dS = 24;$$

$$S_4: \mathbf{F} = \mathbf{0}, \iint_{S_4} \mathbf{F} \cdot d\mathbf{S} = 0;$$

$$S_5: \mathbf{F} = 4x\mathbf{i} + 3xz\mathbf{k}, \mathbf{n} = -\mathbf{j} \text{ and } \iint_{S_5} \mathbf{F} \cdot d\mathbf{S} = \iint_{S_5} 0 \, dS = 0;$$

$$S_6: \mathbf{F} = 4x\mathbf{i} + xy\mathbf{j}, \mathbf{n} = -\mathbf{k} \text{ and } \iint_{S_6} \mathbf{F} \cdot d\mathbf{S} = \iint_{S_6} 0 \, dS = 0.$$

Thus $\iint_S \mathbf{F} \cdot d\mathbf{S} = 64$.

