

Let $u = \ln x, dv = 2x^2 dx \Rightarrow du = dx/x, v = \frac{2}{3}x^3$. Then by equation

$$\int u dv = uv - \int v du,$$

$$\begin{aligned}\int 2x^2 \ln x dx &= \frac{2}{3}x^3 \ln x - \int \frac{2}{3}x^3 (dx/x) = \frac{2}{3}x^3 \ln x - \frac{2}{3} \int x^2 dx \\ &= \frac{2}{3}x^3 \ln x - \frac{2}{3} \cdot \frac{1}{3}x^3 + C = \frac{2}{3}x^3 \ln x - \frac{2}{9}x^3 + C\end{aligned}$$