Let the dimensions be $x$ , $y$ and $z$ , then minimize $xy + 2(xz + yz)$ if $xyz = 42,592$ $\text{cm}^3$ . Then
$f(x, y) = xy + [85,184(x + y)/xy] = xy + 85,184(x^{-1} + y^{-1})$ ,
$f_x = y - 85,184x^{-2}$ , $f_y = x - 85,184y^{-2}$ .
And $f_x = 0$ implies $y = 85,184/x^2$ ; substituting into $f_y = 0$ implies $x^3 = 85,184$ or $x = 44$ and then $y = 44$ . Now
$D(x, y) = [(2)(85,184)]^2 x^{-3} y^{-3} - 1 > 0$ for $(44, 44)$ and $f_{xx}(44, 44) > 0$ so this is indeed a minimum. Thus the dimensions of the box are $x = y = 44$ cm, $z = 22$ cm.