

$$\begin{aligned}
\iint_R \frac{xy^2}{x^2+1} dA &= \int_0^{\textcolor{red}{1}} \int_{-\textcolor{red}{1}}^{\textcolor{red}{1}} \frac{xy^2}{x^2+1} dy dx \\
&= \int_0^{\textcolor{red}{1}} \frac{x}{x^2+1} dx \int_{-\textcolor{red}{1}}^{\textcolor{red}{1}} y^2 dy \\
&= \left[\frac{1}{2} \ln(x^2+1) \right]_0^{\textcolor{red}{1}} \left[\frac{1}{3} y^3 \right]_{-\textcolor{red}{1}}^{\textcolor{red}{1}} \\
&= \frac{1}{2} (\ln(\textcolor{red}{2}) - \ln(1)) \cdot \frac{1}{3} (\textcolor{red}{1} + \textcolor{red}{1}) = (\textcolor{red}{1/3}) \ln(\textcolor{red}{2})
\end{aligned}$$