$$\frac{1}{x(x^2+4)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2} \quad \Rightarrow$$

$1 = A(x^2+4)^2 + (Bx+C)x(x^2+4) + (Dx+E)x.$

Setting $x = 0$ gives $1 = 16A$, so $A = \frac{1}{16}$. Now compare coefficients.

$1 = \frac{1}{16}(x^4 + 8x^2 + 16) + (Bx^2 + Cx)(x^2+4) + Dx^2 + Ex$

$1 = \frac{1}{16}x^4 + \frac{1}{2}x^2 + 1 + Bx^4 + Cx^3 + 4Bx^2 + 4Cx + Dx^2 + Ex$

$1 = \left(\frac{1}{16} + B\right)x^4 + Cx^3 + \left(\frac{1}{2} + 4B + D\right)x^2 + (4C+E)x + 1$

So $B + \frac{1}{16} = 0 \quad \Rightarrow \quad B = -\frac{1}{16}$, $C = 0$, $\frac{1}{2} + 4B + D = 0 \quad \Rightarrow$
$D = -\frac{1}{4}$, and $4C + E = 0 \quad \Rightarrow \quad E = 0$. Thus,

$$\begin{aligned}\int \frac{21\,dx}{x(x^2+4)^2} &= 21\int \left(\frac{\frac{1}{16}}{x} + \frac{-\frac{1}{16}x}{x^2+4} + \frac{-\frac{1}{4}x}{(x^2+4)^2}\right) dx \\ &= \frac{21}{16}\ln|x| - \frac{21}{16}\cdot\frac{1}{2}\ln|x^2+4| - \frac{21}{4}\left(-\frac{1}{2}\right)\frac{1}{x^2+4} + C \\ &= \frac{21}{16}\ln|x| - \frac{21}{32}\ln(x^2+4) + \frac{21}{8}\frac{1}{x^2+4} + C\end{aligned}$$