

$$\begin{aligned}\int 5 \tan^3 x \sec x \, dx &= 5 \int \tan^2 x \sec x \tan x \, dx = 5 \int (\sec^2 x - 1) \sec x \tan x \, dx \\ &= 5 \int (u^2 - 1) \, du \quad [u = \sec x, \, du = \sec x \tan x \, dx] \\ &= \frac{5}{3} u^3 - 5u + C = \frac{5}{3} \sec^3 x - 5 \sec x + C\end{aligned}$$