$t\,\dfrac{dy}{dt} + 4y = t^4,\quad t > 0,\ y(1) = 0.$ Divide by t to get $\dfrac{dy}{dt} + \dfrac{4}{t}\,y = t^3$, which is linear. $I(t) = e^{\int (4/t)\,dt} = e^{4\ln t} = t^4$. Multiplying by t^4 gives $t^4\,\dfrac{dy}{dt} + 4t^3 y = t^7 \quad\Rightarrow\quad (t^4 y)' = t^7 \quad\Rightarrow\quad t^4 y = \frac{1}{8}t^8 + C \quad\Rightarrow\quad y = \dfrac{t^4}{8} + \dfrac{C}{t^4}$. Thus, $0 = y(1) = \frac{1}{8} + C \quad\Rightarrow\quad C = -\frac{1}{8}$, so $y = \dfrac{t^4}{8} - \dfrac{1}{8t^4}$.