$$\int_0^{\pi/2} (4\sin^3 t \, \cos t \, \mathbf{i} + 7\sin t \, \cos^6 t \, \mathbf{j} + 4\sin t \, \cos t \, \mathbf{k}) \, dt$$
$$= \left(\int_0^{\pi/2} 4\sin^3 t \, \cos t \, dt\right) \mathbf{i} + \left(\int_0^{\pi/2} 7\sin t \, \cos^6 t \, dt\right) \mathbf{j} + \left(\int_0^{\pi/2} 4\sin t \, \cos t \, dt\right) \mathbf{k}$$
$$= \left[\sin^4 t\right]_0^{\pi/2} \mathbf{i} + \left[-\cos^7 t\right]_0^{\pi/2} \mathbf{j} + \left[2\sin^2 t\right]_0^{\pi/2} \mathbf{k}$$
$$= (1-0)\,\mathbf{i} + (0+1)\,\mathbf{j} + 2(1-0)\,\mathbf{k} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$$