$r = 2\cos\theta \Rightarrow x = r\cos\theta = 2\cos\theta\cos\theta, \quad y = r\sin\theta = 2\cos\theta\sin\theta \Rightarrow$
$\frac{dy}{d\theta} = -2\sin^2\theta + 2\cos^2\theta = 2\cos 2\theta = 0 \quad \Rightarrow \quad 2\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2} \quad \Leftrightarrow \quad \theta =$
$\frac{\pi}{4}$ or $\frac{3\pi}{4}$.
So the tangent is horizontal at $\left(\frac{2}{\sqrt{2}}, \frac{\pi}{4}\right)$ and $\left(-\frac{2}{\sqrt{2}}, \frac{3\pi}{4}\right)$ $\left[\text{same as } \left(\frac{2}{\sqrt{2}}, -\frac{\pi}{4}\right)\right]$.
$\frac{dx}{d\theta} = -4\sin\theta\cos\theta = -2\sin 2\theta = 0 \Rightarrow 2\theta = 0$ or $\pi \Leftrightarrow \theta = 0$ or $\frac{\pi}{2}$.
So the tangent is vertical at $(2, 0)$ and $\left(0, \frac{\pi}{2}\right)$.