

$\mathbf{r}'(t) = 7t^6 \mathbf{i} + 7t^6 \mathbf{j} + \sqrt{t} \mathbf{k} \Rightarrow \mathbf{r}(t) = t^7 \mathbf{i} + t^7 \mathbf{j} + \frac{2}{3}t^{3/2} \mathbf{k} + \mathbf{C}$, where \mathbf{C} is a constant vector. But $\mathbf{i} + \mathbf{j} = \mathbf{r}(1) = \mathbf{i} + \mathbf{j} + \frac{2}{3}\mathbf{k} + \mathbf{C}$. Thus $\mathbf{C} = -\frac{2}{3}\mathbf{k}$ and $\mathbf{r}(t) = t^7 \mathbf{i} + t^7 \mathbf{j} + (\frac{2}{3}t^{3/2} - \frac{2}{3}) \mathbf{k}$.