

$$f(x, y) = x^3y + 24x^2 - 8y \Rightarrow f_x = 3x^2y + 48x ,$$

$$f_y = x^3 - 8 , \quad f_{xx} = 6xy + 48 , \quad f_{xy} = 3x^2 , \quad f_{yy} = 0 .$$

Then $f_y = 0$ implies $x = 2$, and substitution into $f_x = 0$ gives

$$12y + 96 = 0 \Rightarrow y = -8 . \text{ Thus, the only critical point is } (2, -8) .$$

$$D(2, -8) = (-48)(0) - 12^2 = -144 < 0 , \text{ so } (2, -8) \text{ is a saddle point.}$$

