

$C$  is represented by the vector function

$\mathbf{r}(t) = (t^2 + 1)\mathbf{i} + (t^5 + t)\mathbf{j}$ ,  $0 \leq t \leq 1$ , so  $\mathbf{r}'(t) = 2t\mathbf{i} + (5t^4 + 1)\mathbf{j}$ . Since  $5t^4 + 1 \neq 0$ , we have  $\mathbf{r}'(t) \neq \mathbf{0}$ , thus  $C$  is a smooth curve.  $\nabla f$  is continuous, and hence  $f$  is differentiable, so we have

$$\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(1)) - f(\mathbf{r}(0)) = f(2, 2) - f(1, 0) = 8 - 4 = 4.$$