

$$\begin{aligned}\mathbf{r}(t) &= 8\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k} \quad \Rightarrow \quad \mathbf{r}'(t) = 2t\mathbf{j} + 3t^2\mathbf{k} \quad \Rightarrow \\ |\mathbf{r}'(t)| &= \sqrt{4t^2 + 9t^4} = t\sqrt{4 + 9t^2} \quad [\text{since } t \geq 0]. \quad \text{Then} \\ L &= \int_0^3 |\mathbf{r}'(t)| dt = \int_0^3 t\sqrt{4 + 9t^2} dt = \left[\frac{1}{18} \cdot \frac{2}{3} (4 + 9t^2)^{3/2} \right]_0^3 \\ &= \frac{1}{27} (85^{3/2} - 4^{3/2}) = \frac{1}{27} (85^{3/2} - 8).\end{aligned}$$