

A cross-section is a disk with radius $2 - 2x^2$, so its area is $A(x) = \pi(2 - 2x^2)^2$.

$$\begin{aligned} V &= \int_{-1}^1 A(x) dx = \int_{-1}^1 \pi(2 - 2x^2)^2 dx \\ &= 2\pi \int_0^1 (4 - 8x^2 + 4x^4) dx = 2\pi \left[4x - \frac{8}{3}x^3 + \frac{4}{5}x^5 \right]_0^1 \\ &= 2\pi \left(4 - \frac{8}{3} + \frac{4}{5} \right) = \frac{64}{15}\pi \end{aligned}$$



