Since $f^{(n)}(5) = \dfrac{(-1)^n\, n!}{8^n(n+8)}$, the Taylor series is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(5)}{n!}(x-5)^n = \sum_{n=0}^{\infty} \frac{(-1)^n\, n!}{8^n(n+8)\, n!}(x-5)^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{8^n(n+8)}(x-5)^n,$$

which is the Taylor series for f centered at 5. Apply the Ratio Test to find the radius of convergence R.

$$\begin{aligned}
\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| &= \lim_{n\to\infty}\left|\frac{(-1)^{n+1}(x-5)^{n+1}}{8^{n+1}(n+9)}\cdot\frac{8^n(n+8)}{(-1)^n(x-5)^n}\right| \\
&= \lim_{n\to\infty}\left|\frac{(-1)(x-5)(n+8)}{8(n+9)}\right| = \frac{1}{8}\,|x-5| \lim_{n\to\infty}\frac{n+8}{n+9} \\
&= \frac{1}{8}\,|x-5|
\end{aligned}$$

For convergence, $\frac{1}{8}\,|x-5| < 1 \quad \Leftrightarrow \quad |x-5| < 8$, so $R = 8$.