

$$\begin{aligned} f(x, y) &= 3y^2/x = 3y^2x^{-1} \Rightarrow \\ \nabla f(x, y) &= \langle -3y^2x^{-2}, 6yx^{-1} \rangle = \langle -3y^2/x^2, 6y/x \rangle. \\ \nabla f(2, 4) &= \langle -12, 12 \rangle, \text{ or equivalently } \left\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle, \text{ is the direction of maxi-} \\ &\text{mum rate of change, and the maximum rate is} \\ |\nabla f(2, 4)| &= \sqrt{144 + 144} = 12\sqrt{2}. \end{aligned}$$