$f(x,y)=e^{xy},\ g(x,y)=x^5+y^5=64$, and $\nabla f=\lambda\nabla g \quad\Rightarrow$ $\langle ye^{xy},xe^{xy}\rangle=\langle 5\lambda x^4,5\lambda y^4\rangle$, so $ye^{xy}=5\lambda x^4$ and $xe^{xy}=5\lambda y^4$. Note that $x=0$ $\Leftrightarrow \quad y=0$ which contradicts $x^5+y^5=64$, so we may assume $x\neq 0,\ y\neq 0$, and then $\lambda=ye^{xy}/(5x^4)=xe^{xy}/(5y^4) \quad\Rightarrow\quad x^5=y^5 \quad\Rightarrow\quad x=y$. But $x^5+y^5=64$, so $2x^5=64 \quad\Rightarrow\quad x=2=y$. Here there is no minimum value, since we can choose points satisfying the constraint $x^3+y^3=64$ that make $f(x,y)=e^{xy}$ arbitrarily close to 0 (but never equal to 0). The maximum value is $f(2,2)=e^4$.