

$\operatorname{div}\mathbf{F} = x + y + z$ , so

$$\begin{aligned}\iint\int_E \operatorname{div}\mathbf{F} \, dV &= \int_0^{2\pi} \int_0^8 \int_0^4 (r \cos \theta + r \sin \theta + z) r \, dz \, dr \, d\theta \\ &= \int_0^{2\pi} \int_0^8 4 \left( r^2 \cos \theta + r^2 \sin \theta + \frac{4}{2}r \right) dr \, d\theta \\ &= \int_0^{2\pi} 4 \left( \frac{512}{3} \cos \theta + \frac{512}{3} \sin \theta + \frac{256}{4} \right) d\theta = \frac{1024}{4}(2\pi) = 512\pi\end{aligned}$$

Let  $S_1$  be the top of the cylinder,  $S_2$  the bottom, and  $S_3$  the vertical edge.

On  $S_1$ ,  $z = 4$ ,  $\mathbf{n} = \mathbf{k}$ , and  $\mathbf{F} = xy\mathbf{i} + 4y\mathbf{j} + 4x\mathbf{k}$ , so

$$\begin{aligned}\iint_{S_1} \mathbf{F} \cdot d\mathbf{S} &= \iint_{S_1} \mathbf{F} \cdot \mathbf{n} \, dS = \iint_{S_1} 4x \, dS = \int_0^{2\pi} \int_0^8 4(r \cos \theta) r \, dr \, d\theta \\ &= 4 [\sin \theta]_0^{2\pi} \left[ \frac{1}{3}r^3 \right]_0^8 = 0.\end{aligned}$$

On  $S_2$ ,  $z = 0$ ,  $\mathbf{n} = -\mathbf{k}$ , and  $\mathbf{F} = xy\mathbf{i}$  so  $\iint_{S_2} \mathbf{F} \cdot d\mathbf{S} = \iint_{S_2} 0 \, dS = 0$ .

$S_3$  is given by  $\mathbf{r}(\theta, z) = 8 \cos \theta \mathbf{i} + 8 \sin \theta \mathbf{j} + z \mathbf{k}$ ,  $0 \leq \theta \leq 2\pi$ ,  $0 \leq z \leq 4$ .

Then  $\mathbf{r}_\theta \times \mathbf{r}_z = 8 \cos \theta \mathbf{i} + 8 \sin \theta \mathbf{j}$  and

$$\begin{aligned}\iint_{S_3} \mathbf{F} \cdot d\mathbf{S} &= \iint_D \mathbf{F} \cdot (\mathbf{r}_\theta \times \mathbf{r}_z) \, dA = \int_0^{2\pi} \int_0^4 (64 \cos^2 \theta \sin \theta + 64z \sin^2 \theta) \, dz \, d\theta \\ &= \int_0^{2\pi} \left( 512 \cos^2 \theta \sin \theta + \frac{1024}{2} \sin^2 \theta \right) d\theta \\ &= \left[ -\frac{512}{3} \cos^3 \theta + \frac{1024}{4} \left( \theta - \frac{1}{2} \sin 2\theta \right) \right]_0^{2\pi} = 512\pi\end{aligned}$$

Thus  $\iint_S \mathbf{F} \cdot d\mathbf{S} = 0 + 0 + 512\pi = 512\pi$ .