$$\frac{dz}{dt}+6e^{t+z}=0 \quad \Rightarrow \quad \frac{dz}{dt}=-6e^{t}e^{z} \quad \Rightarrow \quad \int e^{-z}\,dz=-6\int e^{t}\,dt \quad \Rightarrow \quad -e^{-z}=$$
$$-6e^{t}+C \quad \Rightarrow \quad e^{-z}=6e^{t}-C \quad \Rightarrow \quad \frac{1}{e^{z}}=6e^{t}-C \quad \Rightarrow$$
$$e^{z}=\frac{1}{6e^{t}-C} \quad \Rightarrow \quad z=\ln\left(\frac{1}{6e^{t}-C}\right) \quad \Rightarrow \quad z=-\ln(6e^{t}-C)$$