$f(x,y) = (1 + xy)(x + y) = x + y + x^2y + xy^2 \quad \Rightarrow \quad f_x = 1 + 2xy + y^2$, $f_y = 1 + x^2 + 2xy$, $f_{xx} = 2y$, $f_{xy} = 2x + 2y$, $f_{yy} = 2x$. Then $f_x = 0$ implies $1 + 2xy + y^2 = 0$ and $f_y = 0$ implies $1 + x^2 + 2xy = 0$. Subtracting the second equation from the first gives $y^2 - x^2 = 0 \quad \Rightarrow \quad y = \pm x$, but if $y = x$ then $1 + 2xy + y^2 = 0 \quad \Rightarrow \quad 1 + 3x^2 = 0$ which has no real solution. If $y = -x$ then $1 + 2xy + y^2 = 0 \quad \Rightarrow \quad 1 - x^2 = 0$
$\Rightarrow \quad x = \pm 1$, so critical points are $(1, -1)$ and $(-1, 1)$.
$D(1, -1) = (-2)(2) - 0 < 0$ and $D(-1, 1) = (2)(-2) - 0 < 0$, so $(-1, 1)$ and $(1, -1)$ are saddle points.