$f(x, y, z) = 8x - 4z,\ g(x, y, z) = x^2 + 10y^2 + z^2 = 5 \quad \Rightarrow$
$\nabla f = \langle 8, 0, -4 \rangle,\ \lambda \nabla g = \langle 2\lambda x, 20\lambda y, 2\lambda z \rangle$. Then $2\lambda x = 8$,
$20\lambda y = 0,\ 2\lambda z = -4$ imply $x = \dfrac{4}{\lambda},\ y = 0$, and $z = -\dfrac{2}{\lambda}$. But
$5 = x^2 + 10y^2 + z^2 = \left(\dfrac{4}{\lambda}\right)^2 + 10\,(0)^2 + \left(-\dfrac{2}{\lambda}\right)^2 \quad \Rightarrow \quad 5 = \dfrac{20}{\lambda^2} \quad \Rightarrow$
$\lambda = \pm 2$, so f has possible extreme values at the points $(2, 0, -1)$,
$(-2, 0, 1)$. The maximum of f on $x^2 + 10y^2 + z^2 = 5$ is
$f(2, 0, -1) = 20$, and the minimum is $f(-2, 0, 1) = -20$.