If $a_n = (-1)^n \dfrac{n^5 x^n}{3^n}$, then $\lim\limits_{n\to\infty} \left| \dfrac{a_{n+1}}{a_n} \right| = \lim\limits_{n\to\infty} \left| \dfrac{(n+1)^5\, x^{n+1}}{3^{n+1}} \cdot \dfrac{3^n}{n^5\, x^n} \right|$
$= \lim\limits_{n\to\infty} \left| \dfrac{x(n+1)^5}{3n^5} \right| = \lim\limits_{n\to\infty} \left[\dfrac{|x|}{3} \left(1 + \dfrac{1}{n} \right)^5 \right] = \dfrac{|x|}{3} (1)^5 = \frac{1}{3}\, |x|$.

By the Ratio Test, the series $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{n^5 x^n}{3^n}$ converges when $\frac{1}{3}\, |x| < 1 \quad \Leftrightarrow \quad |x| < 3$, so the radius of convergence is $R = 3$. When $x = \pm 3$, both series $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{n^5 (\pm 3)^n}{3^n} = \sum\limits_{n=1}^{\infty} (\mp 1)^n n^5$ diverge by the Test for Divergence since $\lim\limits_{n\to\infty} |(\mp 1)^n\, n^5| = \infty$. Thus, the interval of convergence is $I = (-3, 3)$.