$$
\begin{aligned}
L &= \int_a^b \sqrt{r^2 + (dr/d\theta)^2}\, d\theta = \int_0^{\frac{3\pi}{2}} \sqrt{(\theta^2)^2 + (2\theta)^2}\, d\theta \\
&= \int_0^{\frac{3\pi}{2}} \sqrt{\theta^4 + 4\theta^2}\, d\theta = \int_0^{\frac{3\pi}{2}} \sqrt{\theta^2(\theta^2 + 4)}\, d\theta \\
&= \int_0^{\frac{3\pi}{2}} \theta\sqrt{\theta^2 + 4}\, d\theta
\end{aligned}
$$

Now let $u = \theta^2 + 4$, so that $du = 2\theta\, d\theta \quad \left[\theta\, d\theta = \frac{1}{2}\, du\right]$ and

$$
\int_0^{\frac{3\pi}{2}} \theta\sqrt{\theta^2 + 4}\, d\theta = \int_4^{\frac{9\pi^2}{4}+4} \tfrac{1}{2}\sqrt{u}\, du = \tfrac{1}{2} \cdot \tfrac{2}{3}\left[u^{3/2}\right]_4^{\frac{9\pi^2}{4}+4} = \frac{(\frac{9\pi^2}{4} + 4)^{3/2} - 8}{3}
$$