$$y = \ln(1-x^2) \quad \Rightarrow \quad y' = \frac{1}{1-x^2}\cdot(-2x) \quad \Rightarrow \quad 1+\left(\frac{dy}{dx}\right)^2$$
$$= 1+\frac{4x^2}{(1-x^2)^2} = \frac{1-2x^2+x^4+4x^2}{(1-x^2)^2} = \frac{1+2x^2+x^4}{(1-x^2)^2} = \frac{(1+x^2)^2}{(1-x^2)^2} \quad \Rightarrow$$
$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{\left(\frac{1+x^2}{1-x^2}\right)^2} = \frac{1+x^2}{1-x^2} = -1+\frac{2}{1-x^2} \quad [\text{by division}] \quad = -1+$$
$$\frac{1}{1+x}+\frac{1}{1-x} \quad [\text{partial fractions}].$$
So $L = \int_0^{1/6}\left(-1+\frac{1}{1+x}+\frac{1}{1-x}\right)dx = \left[-x+\ln|1+x|-\ln|1-x|\right]_0^{1/6}$
$= \left(-\frac{1}{6}+\ln\frac{7}{6}-\ln\frac{5}{6}\right)-0 = \ln\left(\frac{7}{5}\right)-\frac{1}{6}$.