$b_n = \dfrac{n}{\sqrt{n^3+6}} > 0$ for $n \geq 1$. $\{b_n\}$ is decreasing for $n \geq 3$ since

$$\begin{aligned}
\left(\frac{x}{\sqrt{x^3+6}}\right)' &= \frac{(x^3+6)^{1/2}(1) - x\cdot\frac{1}{2}(x^3+6)^{-1/2}(3x^2)}{\left(\sqrt{x^3+6}\right)^2} \\
&= \frac{\frac{1}{2}(x^3+6)^{-1/2}[2(x^3+6)-3x^3]}{(x^3+6)^1} \\
&= \frac{12-x^3}{2(x^3+6)^{3/2}} < 0 \text{ for } x > \sqrt[3]{12} \approx 2.3\,.
\end{aligned}$$

Also, $\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} \dfrac{n/n}{\sqrt{n^3+6}/\sqrt{n^2}} = \lim\limits_{n\to\infty} \dfrac{1}{\sqrt{n+6/n^2}} = 0$. Thus, the series $\sum\limits_{n=1}^{\infty} (-1)^n \dfrac{n}{\sqrt{n^3+6}}$ converges by the Alternating Series Test.