$f(x, y, z) = xe^{4yz}$

(a) $\nabla f(x, y, z) = \langle f_x(x, y, z), f_y(x, y, z), f_z(x, y, z) \rangle = \langle e^{4yz}, 4xze^{4yz}, 4xye^{4yz} \rangle$

(b) $\nabla f(2, 0, 1) = \langle 1, 8, 0 \rangle$

(c) By equation $D_{\mathbf{u}}f(x, y, z) = \nabla f(x, y, z) \cdot \mathbf{u}$,

$$D_{\mathbf{u}}f(2, 0, 1) \quad = \nabla f(2, 0, 1) \cdot \mathbf{u} = \langle 1, 8, 0 \rangle \cdot \langle \tfrac{2}{3}, -\tfrac{2}{3}, \tfrac{1}{3} \rangle = \tfrac{2}{3} - \tfrac{16}{3} + 0$$
$$= -\tfrac{14}{3}.$$