$f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2 + 7 \Rightarrow \quad f_x = 6x^2 + y^2 + 10x$,

$f_y = 2xy + 2y$, $f_{xx} = 12x + 10$, $f_{yy} = 2x + 2$, $f_{xy} = 2y$. Then

$f_y = 0$ implies $y = 0$ or $x = -1$. Substituting into $f_x = 0$ gives the

critical points $(0,0)$, $\left(-\frac{5}{3}, 0\right)$, $(-1, \pm 2)$. Now $D(0,0) = 20 > 0$

and $f_{xx}(0,0) = 10 > 0$, so $f(0,0) = 7$ is a local minimum.

Also $f_{xx}\left(-\frac{5}{3}, 0\right) < 0$, $D\left(-\frac{5}{3}, 0\right) > 0$, and $D(-1, \pm 2) < 0$.

Hence $f\left(-\frac{5}{3}, 0\right) = \frac{314}{27}$ is a local maximum while $(-1, \pm 2)$ are saddle points.