$f(x, y, z) = x^4 + y^4 + z^4$, $g(x, y, z) = x^8 + y^8 + z^8 = 1 \quad \Rightarrow$
$\nabla f = \langle 4x^3, 4y^3, 4z^3 \rangle$, $\lambda \nabla g = \langle 8\lambda x^7, 8\lambda y^7, 8\lambda z^7 \rangle$.

Case 1: If $x \neq 0$, $y \neq 0$ and $z \neq 0$, then $\nabla f = \lambda \nabla g$ implies $\lambda = 1/(2x^4) = 1/(2y^4) = 1/(2z^4)$ or $x^4 = y^4 = z^4$ and $3x^8 = 1$ or $x = \pm\frac{1}{\sqrt[8]{3}}$ giving the points $\left(\pm\frac{1}{\sqrt[8]{3}}, \frac{1}{\sqrt[8]{3}}, \frac{1}{\sqrt[8]{3}}\right)$, $\left(\pm\frac{1}{\sqrt[8]{3}}, -\frac{1}{\sqrt[8]{3}}, \frac{1}{\sqrt[8]{3}}\right)$, $\left(\pm\frac{1}{\sqrt[8]{3}}, \frac{1}{\sqrt[8]{3}}, -\frac{1}{\sqrt[8]{3}}\right)$, $\left(\pm\frac{1}{\sqrt[8]{3}}, -\frac{1}{\sqrt[8]{3}}, -\frac{1}{\sqrt[8]{3}}\right)$ all with an f-value of $\sqrt{3}$.

Case 2: If one of the variables equals zero and the other two are not zero, then the fourth powers of the two nonzero coordinates are equal with common value $\frac{1}{\sqrt{2}}$ and corresponding f value of $\sqrt{2}$.

Case 3: If exactly two of the variables are zero, then the third variable has value ± 1 with the corresponding f value of 1. Thus on $x^8 + y^8 + z^8 = 1$, the maximum value of f is $\sqrt{3}$ and the minimum value is 1.