$\iint_R ye^x\,dA = \int_0^{\pi/2}\int_0^{5}(r\sin\theta)\,e^{r\cos\theta}\,r\,dr\,d\theta = \int_0^{5}\int_0^{\pi/2} r^2\sin\theta\,e^{r\cos\theta}\,d\theta\,dr$. First we integrate
$\int_0^{\pi/2} r^2\sin\theta\,e^{r\cos\theta}\,d\theta$: Let $u = r\cos\theta \quad \Rightarrow \quad du = -r\sin\theta\,d\theta$, and
$\int_0^{\pi/2} r^2\sin\theta\,e^{r\cos\theta}\,d\theta = \int_{u=r}^{u=0} -r\,e^u\,du = -r[e^0 - e^r] = re^r - r$.
Then $\int_0^{5}\int_0^{\pi/2} r^2\sin\theta\,e^{r\cos\theta}\,d\theta\,dr = \int_0^{5}(re^r - r)\,dr = \left[re^r - e^r - \frac{1}{2}r^2\right]_0^{5} = 4e^{5} - \dfrac{23}{2}$, where we integrated by parts in the first term.