

$$(a) \quad \operatorname{curl} \mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \ln x & \ln(xy) & \ln(xyz) \end{vmatrix} \\ = \left( \frac{xz}{xyz} - 0 \right) \mathbf{i} - \left( \frac{yz}{xyz} - 0 \right) \mathbf{j} + \left( \frac{y}{xy} - 0 \right) \mathbf{k} = \left\langle \frac{1}{y}, -\frac{1}{x}, \frac{1}{x} \right\rangle$$

$$(b) \quad \operatorname{div} \mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial}{\partial x} (\ln x) + \frac{\partial}{\partial y} (\ln(xy)) + \frac{\partial}{\partial z} (\ln(xyz)) \\ = \frac{1}{x} + \frac{x}{xy} + \frac{xy}{xyz} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$