(a) $$f(x) = \frac{1}{(2+x)^2} = \frac{d}{dx}\left(1/2\frac{-1}{1+x/2}\right)$$
$$= -1/2\frac{d}{dx}\left[\sum_{n=0}^{\infty}(-1)^n\,(x/2)^n\right]$$
$$= 1/2\sum_{n=1}^{\infty}(-1)^{n+1}n(x/2)^{n-1}(1/2)$$
$$= \sum_{n=0}^{\infty}(-1)^n(n+1)x^n/2^{n+2} \text{ with } R = 2\ .$$

In the last step, note that we *decreased* the initial value of the summation variable n by 1 , and then *increased* each occurrence of n in the term by 1 [also note that $(-1)^{n+2} = (-1)^n$].

(b) $$f(x) = \frac{1}{(2+x)^3} = -\frac{1}{2}\frac{d}{dx}\left[\frac{1}{(2+x)^2}\right]$$
$$= -\frac{1}{2}\frac{d}{dx}\left[1/4\sum_{n=0}^{\infty}(-1)^n(n+1)(x/2)^n\right] \quad \text{[from part (a)]}$$
$$= -\tfrac{1}{8}\sum_{n=1}^{\infty}(-1)^n(n+1)n(x/2)^{n-1}(1/2)$$
$$= \tfrac{1}{2}\sum_{n=0}^{\infty}(-1)^n(n+2)(n+1)x^n/2^{n+3} \text{ with } R = 2\ .$$

(c) $$f(x) = \frac{x^2}{(2+x)^3} = x^2\cdot\frac{1}{(2+x)^3}$$
$$= x^2\cdot\frac{1}{2}\sum_{n=0}^{\infty}(-1)^n(n+2)(n+1)x^n/2^{n+3} \quad \text{[from part (b)]}$$
$$= \frac{1}{2}\sum_{n=0}^{\infty}(-1)^n(n+2)(n+1)x^{n+2}/2^{n+3}$$

To write the power series with x^n rather than x^{n+2}, we will *decrease* each occurrence of n in the term by 2 and *increase* the initial value of the summation variable by 2. This gives us $\frac{1}{2}\sum_{n=2}^{\infty}(-1)^n(n)(n-1)x^n/2^{n+1}$ with $R = 2$.