$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^n \frac{7n-3}{4n+3} = \sum_{n=1}^{\infty} (-1)^n b_n.$$

Now $\lim\limits_{n \to \infty} b_n = \lim\limits_{n \to \infty} \dfrac{7 - 3/n}{4 + 3/n} = \dfrac{7}{4} \neq 0$. Since $\lim\limits_{n \to \infty} a_n \neq 0$ (in fact the limit does not exist), the series diverges by the Test for Divergence.