

$$f(x, y, z) = \sqrt{x + yz} = (x + yz)^{1/2}$$

$$\begin{aligned} \text{(a)} \quad \nabla f(x, y, z) &= \left\langle \frac{1}{2}(x + yz)^{-1/2}, \frac{1}{2}(x + yz)^{-1/2}(z), \frac{1}{2}(x + yz)^{-1/2}(y) \right\rangle \\ &= \left\langle 1/(2\sqrt{x + yz}), z/(2\sqrt{x + yz}), y/(2\sqrt{x + yz}) \right\rangle \end{aligned}$$

$$\text{(b)} \quad \nabla f(\mathbf{3}, \mathbf{1}, \mathbf{1}) = \left\langle \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \right\rangle$$

$$\begin{aligned} \text{(c)} \quad D_{\mathbf{u}}f(\mathbf{3}, \mathbf{1}, \mathbf{1}) &= \nabla f(\mathbf{3}, \mathbf{1}, \mathbf{1}) \cdot \mathbf{u} = \left\langle \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \right\rangle \cdot \left\langle \frac{3}{7}, \frac{6}{7}, \frac{2}{7} \right\rangle \\ &= \frac{3}{28} + \frac{6}{28} + \frac{2}{28} = \frac{11}{28} \end{aligned}$$