(a) $f_x(x,y) = y^2/(1+x^2)$ implies $f(x,y) = y^2 \arctan x + g(y) \;\Rightarrow$
$f_y(x,y) = 2y \arctan x + g'(y)$. But $f_y(x,y) = 2y \arctan x$ so $g'(y) = 0 \;\Rightarrow\; g(y) = K$. We can take $K = 0$, so
$f(x,y) = y^2 \arctan x$.

(b) The initial point of C is $\mathbf{r}(0) = (0,0)$ and the terminal point is $\mathbf{r}(1) = (1,8)$, so
$\int_C \mathbf{F} \cdot d\,\mathbf{r} = f(1,8) - f(0,0) = 64 \arctan 1 - 0 = 64 \cdot \frac{\pi}{4} = 16\pi$.