

Let $u = \ln(7x + 1)$, $dv = dx \Rightarrow du = \frac{7}{7x + 1} dx$, $v = x$. Then

$$\begin{aligned}\int \ln(7x + 1) dx &= x \ln(7x + 1) - \int \frac{7x}{7x + 1} dx = x \ln(7x + 1) - \int \frac{(7x + 1) - 1}{7x + 1} dx \\&= x \ln(7x + 1) - \int \left(1 - \frac{1}{7x + 1}\right) dx \\&= x \ln(7x + 1) - x + \frac{1}{7} \ln(7x + 1) + C \\&= \frac{1}{7}(7x + 1) \ln(7x + 1) - x + C.\end{aligned}$$