

$n$	$f^{(n)}(x)$	$f^{(n)}(5)$
0	$\ln x$	$\ln 5$
1	$1/x$	$1/5$
2	$-1/x^2$	$-1/5^2$
3	$2/x^3$	$2/5^3$
4	$-6/x^4$	$-6/5^4$
5	$24/x^5$	$24/5^5$
$\vdots$	$\vdots$	$\vdots$

$$\begin{aligned}
f(x) &= \ln x = \sum_{n=0}^{\infty} \frac{f^{(n)}(5)}{n!} (x-5)^n \\
&= \frac{\ln 5}{0!} (x-5)^0 + \frac{1}{1!5^1} (x-5)^1 + \frac{-1}{2!5^2} (x-5)^2 + \frac{2}{3!5^3} (x-5)^3 \\
&\quad + \frac{-6}{4!5^4} (x-5)^4 + \frac{24}{5!5^5} (x-5)^5 + \dots \\
&= \ln 5 + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(n-1)!}{n!5^n} (x-5)^n \\
&= \ln 5 + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n5^n} (x-5)^n \\
\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| &= \lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+2} (x-5)^{n+1}}{(n+1)5^{n+1}} \cdot \frac{n5^n}{(-1)^{n+1} (x-5)^n} \right| \\
&= \lim_{n \rightarrow \infty} \left| \frac{(-1)(x-5)n}{(n+1)5} \right| \\
&= \lim_{n \rightarrow \infty} \left( \frac{n}{n+1} \right) \frac{|x-5|}{5} \\
&= \frac{|x-5|}{5} < 1 \text{ for convergence, so } |x-5| < 5 \text{ and } R = 5.
\end{aligned}$$