$r = 2\tan\theta\sec\theta = \dfrac{2\sin\theta}{\cos^2\theta} \Rightarrow r\cos^2\theta = 2\sin\theta \Leftrightarrow (r\cos\theta)^2 = 2r\sin\theta \Leftrightarrow x^2 = 2y,$
a parabola with vertex at the origin opening upward.
The first implication is reversible since $\cos\theta = 0$ would imply
$2\sin\theta = r\cos^2\theta = 0$, contradicting the fact that $\cos^2\theta + \sin^2\theta = 1$.