

$y' + 5y = \sin(e^{5x})$, so $I(x) = e^{\int P(x) dx} = e^{\int 5 dx} = e^{5x}$. Multiplying the differential equation by $I(x)$ gives $e^{5x}y' + 5e^{5x}y = e^{5x} \sin(e^{5x}) \Rightarrow (e^{5x}y)' = e^{5x} \sin(e^{5x}) \Rightarrow e^{5x}y = \int e^{5x} \sin(e^{5x}) dx \Rightarrow e^{5x}y = -(1/5) \cos(e^{5x}) + C \Rightarrow y = -(1/5)e^{-5x} \cos(e^{5x}) + Ce^{-5x}$.