If $p > -2$, $\dfrac{1}{(n+1)^{p+2}} \leq \dfrac{1}{n^{p+2}}$ ($\{1/n^{p+2}\}$ is decreasing) and $\lim\limits_{n\to\infty} \dfrac{1}{n^{p+2}} = 0$, so the series converges by the Alternating Series Test. If $p \leq -2$, $\lim\limits_{n\to\infty} \dfrac{(-1)^{n-1}}{n^{p+2}}$ does not exist, so the series diverges by the Test for Divergence. Thus, $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^{n-1}}{n^{p+2}}$ converges $\Leftrightarrow$ $p > -2$.