

The paraboloid $z = 1 - x^2 - y^2$ intersects the xy -plane in the circle $x^2 + y^2 = r^2 = 1$ or $r = 1$, so in cylindrical coordinates, E is given by $\{(r, \theta, z) \mid 0 \leq \theta \leq \frac{\pi}{2}, 0 \leq r \leq 1, 0 \leq z \leq 1 - r^2\}$.

Thus

$$\begin{aligned} \iiint_E 3(x^3 + xy^2) dV &= 3 \int_0^{\pi/2} \int_0^1 \int_0^{1-r^2} (r^3 \cos^3 \theta + r^3 \cos \theta \sin^2 \theta) r dz dr d\theta = 3 \int_0^{\pi/2} \int_0^1 \int_0^{1-r^2} r^4 \cos \theta dz dr d\theta \\ &= 3 \int_0^{\pi/2} \int_0^1 r^4 \cos \theta [z]_{z=0}^{z=1-r^2} dr d\theta = 3 \int_0^{\pi/2} \int_0^1 r^4 (1 - r^2) \cos \theta dr d\theta \\ &= 3 \int_0^{\pi/2} \cos \theta \left[\frac{1}{5} r^5 - \frac{1}{7} r^7 \right]_{r=0}^{r=1} d\theta = \int_0^{\pi/2} \frac{6}{35} \cos \theta d\theta = \frac{6}{35} [\sin \theta]_0^{\pi/2} = \frac{6}{35} \end{aligned}$$