

(a) $f_x(x, y, z) = yz$ implies $f(x, y, z) = xyz + g(y, z)$ and so
 $f_y(x, y, z) = xz + g_y(y, z)$. But $f_y(x, y, z) = xz$ so
 $g_y(y, z) = 0 \Rightarrow g(y, z) = h(z)$. Thus $f(x, y, z) = xyz + h(z)$ and
 $f_z(x, y, z) = xy + h'(z)$. But $f_z(x, y, z) = xy + 2z$, so
 $h'(z) = 2z \Rightarrow h(z) = z^2 + K$.
Hence $f(x, y, z) = xyz + z^2$ (taking $K = 0$).

(b) $\int_C \mathbf{F} \cdot d\mathbf{r} = f(5, 6, 2) - f(2, 0, -2) = 64 - 4 = 60$.