

By Green's Theorem, $W = \int_C \mathbf{F} \cdot d\mathbf{r} = \int_C x(x+y) dx + xy^2 dy$
 $= \iint_D (y^2 - x) dy dx$ where C is the path described in the question and D is
the triangle bounded by C . So

$$\begin{aligned} W &= \int_0^6 \int_0^{6-x} (y^2 - x) dy dx = \int_0^6 \left[\frac{1}{3}y^3 - xy \right]_{y=0}^{y=6-x} dx \\ &= \int_0^6 \left(\frac{1}{3}(6-x)^3 - x(6-x) \right) dx \\ &= \left[-\frac{1}{12}(6-x)^4 - 3x^2 + \frac{1}{3}x^3 \right]_0^6 \\ &= (-108 + 72) - (-108) = 72 \end{aligned}$$