

$\operatorname{div} \mathbf{F} = y^2 + 0 + x^2 = x^2 + y^2$  so

$$\begin{aligned} \iint_S \mathbf{F} \cdot d\mathbf{S} &= \iiint_E (x^2 + y^2) dV = \int_0^{2\pi} \int_0^1 \int_{r^2}^1 r^2 \cdot r \, dz \, dr \, d\theta \\ &= \int_0^{2\pi} \int_0^1 r^3(1 - r^2) \, dr \, d\theta = \int_0^{2\pi} d\theta \int_0^1 (r^3 - r^5) \, dr \\ &= 2\pi \left[ \frac{1}{4}r^4 - \frac{1}{6}r^6 \right]_0^1 = \frac{1}{6}\pi \end{aligned}$$