$$\frac{x^2 - x + 18}{x^3 + 3x} = \frac{x^2 - x + 18}{x(x^2 + 3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3}.$$ Multiply by $x(x^2 + 3)$ to get $x^2 - x + 18 = A(x^2 + 3) + (Bx + C)x$. Substituting 0 for x gives $18 = 3A \Leftrightarrow A = 6$. The coefficients of the x^2-terms must be equal, so $1 = A + B \Rightarrow B = 1 - 6 = -5$. The coefficients of the x-terms must be equal, so $-1 = C$. Thus,

$$\int \frac{x^2 - x + 18}{x^3 + 3x}\, dx \quad = \int \left(\frac{6}{x} + \frac{-5x - 1}{x^2 + 3} \right) dx = \int \left(\frac{6}{x} - 5\frac{x}{x^2 + 3} - \frac{1}{x^2 + 3} \right) dx$$

$$= 6\ln|x| - \frac{5}{2}\ln(x^2 + 3) - \frac{1}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} + C$$