

A cross-section is a washer with inner radius $\frac{1}{2}y^2$ and outer radius $2y$, so its area is $A(y) = \pi(2y)^2 - \pi(\frac{1}{2}y^2)^2 = \pi(4y^2 - \frac{1}{4}y^4)$.

$$\begin{aligned} V &= \int_0^4 A(y) dy = \pi \int_0^4 (4y^2 - \frac{1}{4}y^4) dy \\ &= \pi [\frac{4}{3}y^3 - \frac{1}{20}y^5]_0^4 = \pi (\frac{256}{3} - \frac{256}{5}) = \frac{512}{15}\pi \end{aligned}$$

