$\mathbf{a}(t) = 4t\,\mathbf{i} + \sin t\,\mathbf{j} + \cos 2t\,\mathbf{k} \quad \Rightarrow$

$\mathbf{v}(t) = \int (4t\,\mathbf{i} + \sin t\,\mathbf{j} + \cos 2t\,\mathbf{k})\,dt = 2t^2\,\mathbf{i} - \cos t\,\mathbf{j} + \frac{1}{2}\sin 2t\,\mathbf{k} + \mathbf{C}$

and $\mathbf{i} = \mathbf{v}\,(0) = -\mathbf{j} + \mathbf{C}$, so $\mathbf{C} = \mathbf{i} + \mathbf{j}$

and $\mathbf{v}(t) = (2t^2 + 1)\,\mathbf{i} + (1 - \cos t)\,\mathbf{j} + \frac{1}{2}\sin 2t\,\mathbf{k}$.

$$\begin{aligned}\mathbf{r}(t) &= \int [(2t^2+1)\,\mathbf{i} + (1-\cos t)\,\mathbf{j} + \tfrac{1}{2}\sin 2t\,\mathbf{k}]dt \\ &= \left(\tfrac{2}{3}t^3 + t\right)\,\mathbf{i} + (t - \sin t)\,\mathbf{j} - \tfrac{1}{4}\cos 2t\,\mathbf{k} + \mathbf{D}\end{aligned}$$

But $\mathbf{j} = \mathbf{r}\,(0) = -\frac{1}{4}\mathbf{k} + \mathbf{D}$, so $\mathbf{D} = \mathbf{j} + \frac{1}{4}\mathbf{k}$ and

$\mathbf{r}(t) = \left(\frac{2}{3}t^3 + t\right)\mathbf{i} + (t - \sin t + 1)\,\mathbf{j} + \left(\frac{1}{4} - \frac{1}{4}\cos 2t\right)\mathbf{k}$.