First let $u = \sin(5\theta)$, $dv = e^{4\theta}\, d\theta \quad \Rightarrow \quad du = 5\cos(5\theta)\, d\theta$, $v = \frac{1}{4}e^{4\theta}$.
Then $I = \int e^{4\theta}\sin(5\theta)\, d\theta = \frac{1}{4}e^{4\theta}\sin(5\theta) - \frac{5}{4}\int e^{4\theta}\cos(5\theta)\, d\theta$.
Next let $U = \cos(5\theta)$, $dV = e^{4\theta}\, d\theta \quad \Rightarrow \quad dU = -5\sin(5\theta)\, d\theta$, $V = \frac{1}{4}e^{4\theta}$ to get $\int e^{4\theta}\cos(5\theta)\, d\theta = \frac{1}{4}e^{4\theta}\cos(5\theta) + \frac{5}{4}\int e^{4\theta}\sin(5\theta)\, d\theta$.
Substituting in the previous formula gives
$I = \frac{1}{4}e^{4\theta}\sin(5\theta) - \frac{5}{16}e^{4\theta}\cos(5\theta) - \frac{25}{16}\int e^{4\theta}\sin(5\theta)\, d\theta$
$= \frac{1}{4}e^{4\theta}\sin(5\theta) - \frac{5}{16}e^{4\theta}\cos(5\theta) - \frac{25}{16}I \quad \Rightarrow$
$\frac{41}{16}I = \frac{1}{4}e^{4\theta}\sin(5\theta) - \frac{5}{16}e^{4\theta}\cos(5\theta) + C_1$.
Hence, $I = \frac{1}{41}e^{4\theta}(4\sin(5\theta) - 5\cos(5\theta)) + C$, where $C = \frac{16}{41}C_1$.